Assignment 5

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Attempt all questions.

Q.1: Discriminator Gain

A loop discriminator is an odd function of the input error and, for small errors, can be approximated by a constant gain:

$$D(\Delta\tau) = f_{odd}(\Delta\tau) \approx G_d \Delta\tau + O(\Delta\tau^3). \tag{1}$$

The discriminator gain is defined as

$$G_d = \left. \frac{\partial \mathrm{E}\left[D(\Delta\tau)\right]}{\partial(\Delta\tau)} \right|_{\Delta\tau=0} \tag{2}$$

where the expected value operator, $\mathrm{E}[\cdot]$, is used to remove the impact of the noise at the input of the discriminator. It can be shown that (2) is equivalent to

$$G_d = \left. \frac{\partial \bar{D}(\Delta\tau)}{\partial(\Delta\tau)} \right|_{\Delta\tau=0} \tag{3}$$

where $\bar{D}(\Delta\tau)$ is the discriminator output obtained in the absence of input noise.
Consider the normalized non-coherent early-minus-late power code discriminator:

$$D(\Delta\tau) = \frac{|E|^2 - |L|^2}{|E|^2 + |L|^2} \tag{4}$$

where E and L are the complex early and late correlator outputs. In the absence of input noise

$$\begin{aligned} |E| &= AR\left(\Delta\tau - d_s/2\right) \\ |L| &= AR\left(\Delta\tau + d_s/2\right) \end{aligned} \tag{5}$$

where $R(\cdot)$ is the signal correlation function, A is the correlator amplitude and d_s is the early-minus-late spacing.
Determine the discriminator gain for the normalized non-coherent early-minus-late power code discriminator (4) as a function of the correlation $R(\cdot)$ and its derivative $R'(\cdot)$.

Hints:

- the following chain rule can be useful

$$\frac{\partial \bar{D}(\Delta\tau)}{\partial(\Delta\tau)} = \frac{\partial \bar{D}(\Delta\tau)}{\partial |E|} \cdot \frac{\partial |E|}{(\Delta\tau)} + \frac{\partial \bar{D}(\Delta\tau)}{\partial |L|} \cdot \frac{\partial |L|}{(\Delta\tau)}. \tag{6}$$

- $R(\cdot)$ can be assumed to be even ($R(\Delta\tau) = R(-\Delta\tau)$).
- $R'(\cdot)$ can be assumed to be odd ($R'(\Delta\tau) = -R'(-\Delta\tau)$).

Evaluate the discriminator gain for:

- an ideal BPSK modulation

$$R(\Delta\tau) = \begin{cases} 1 - |\Delta\tau| & |\Delta\tau| < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

- an ideal BOC(1, 1) modulation

$$R(\Delta\tau) = \begin{cases} 1 - 3|\Delta\tau| & |\Delta\tau| < 0.5 \\ -1 + |\Delta\tau| & 0.5 < |\Delta\tau| < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Remark: $\Delta\tau$ and d_s are in units of code chips.

Q.2: S-Curve Analysis
The following Matlab m-files have been provided with this assignment:

- GpsL1CAIdealCorrelation.m - function that evaluates the correlation of a BPSK signal as a function of the code delay in units of chips;
- QuasiCoherentScurve.m - function that evaluates the S-curve for a quasi-coherent discriminator for a Gps L1 C/A signal (unit amplitude);
- SCurveExample.m - example of use of the previous two functions.

Use the previous functions as a model for the evaluation of the S-curve of

- coherent early-minus-late discriminator;
- non-coherent early-minus-late envelope discriminator.

Plot the S-curve of the mentioned discriminators for different early-minus-late spacing:

$$d_s = [0.1, 0.25, 0.5, 1] \qquad \text{(units of chips).}$$

What is the impact of the early-minus-late spacing on the S-curve? What are desirable properties for an S-curve?