Assignment 3

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Attempt all questions.

Q.1: I/Q sampling
When dealing with Radio Frequency (RF) signals, it is possible to adopt several down-conversion/sampling scheme. An example is given in Fig. 1 where a real RF signal is down-converted and sampled using two orthogonal sinusoids. This sampling scheme is called In-phase/Quadrature (I/Q) sampling and allows one to recover a digital base-band version of the analog signal at the receiver antenna. Assume that the signal at the front-end antenna can be modeled as

$$y(t) = b(t)\cos(2\pi f_0 t + \phi) \tag{1}$$

where $b(t)$ is a base-band signal with PSD different from zero in the frequency interval $[-B/2; B/2]$.
1) Provide a graphical representation of the PSD of $y(t)$.
2) $y(t)$ is recovered by the receiver antenna and multiplied by two orthogonal sinusoids:

$$\begin{aligned} y_I(t) &= y(t)\cos(2\pi f_{loc} t) \\ y_Q(t) &= -y(t)\sin(2\pi f_{loc} t). \end{aligned} \tag{2}$$

f_{loc} is the frequency of the local carriers and it is chosen such that $|f_0 - f_{loc}|$ is close to zero. $y_I(t)$ and $y_Q(t)$ are called the in-phase and quadrature components of the base-band version of $y(t)$. Determine and sketch

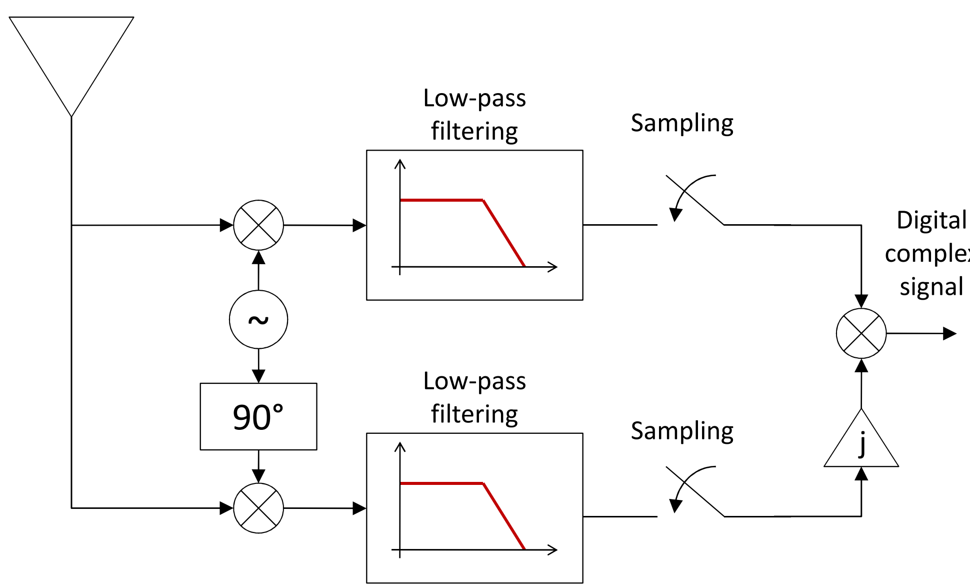

Figure 1: I/Q sampling scheme.

the PSD of $y_I(t)$ and $y_Q(t)$. **3)** $y_I(t)$ and $y_Q(t)$ are then low-pass filtered. It is assumed that the filters in Fig. 1 completely preserve the low frequency components of $y_I(t)$ and $y_Q(t)$ and remove the signals centered at $\pm(f_0 + f_{loc})$. The filtered version of $y_I(t)$ and $y_Q(t)$, $\tilde{y}_I(t)$ and $\tilde{y}_Q(t)$ are sampled with the sampling frequency $f_s = \frac{1}{T_s}$. Evaluate ans sketch the Fourier transforms of

$$\begin{aligned} \tilde{y}_I(nT_s) &= \tilde{y}_I(t) \cdot \sum_{i=-\infty}^{+\infty} \delta\left(t - nT_s\right) \\ \tilde{y}_Q(nT_s) &= \tilde{y}_Q(t) \cdot \sum_{i=-\infty}^{+\infty} \delta\left(t - nT_s\right) \end{aligned} \tag{3}$$

4) The final complex base-band digital signal is obtained as $\tilde{b}(nT_s) = \tilde{y}_I(nT_s) + j\tilde{y}_Q(nT_s)$. Determine $\tilde{b}(nT_s)$ and its Fourier Transform.
5) What is the minimum sampling frequency, f_s, that can adopted for sampling $\tilde{y}_I(t)$ and $\tilde{y}_Q(t)$? What happen if $f_{loc} = f_0$?
6) In the case where $y(t)$ is affected by a non-negligible Doppler shift, is it possible to use this sampling scheme with $f_{loc} = f_0$ for estimating the signal Doppler frequency unambiguously?
Hint: Express the different Fourier Transforms as a function of $B(f)$, the Fourier transform of $b(t)$.

Q.2: I/Q imbalance

A common manufacturing fault in front-ends adopting I/Q sampling is the so called **I/Q imbalance**. When I/Q imbalance is present, the in-phase and quadrature components, $\tilde{y}_I[n] = \tilde{y}_I(nT_s)$ and $\tilde{y}_Q[n] = \tilde{y}_Q(nT_s)$, are scaled differently and the final recovered signal is given by:

$$\tilde{b}_{im}[n] = a\tilde{y}_I[n] + jb\tilde{y}_Q[n] \tag{4}$$

where a and b are two constants.
1) Show that $\tilde{b}_{im}[n]$ can be expressed as

$$\tilde{b}_{im}[n] = \frac{a+b}{2}\tilde{b}[n] + \frac{a-b}{2}\tilde{b}^*[n] \tag{5}$$

where $\tilde{b}[n] = \tilde{y}_I[n] + j\tilde{y}_q[n]$ and $*$ denotes complex conjugate.
2) Consider the case where the recovered signal, $\tilde{b}[n]$, is a complex sinusoid at the frequency f_b. Show that I/Q imbalance generates an interfering term at the frequency $-f_b$.
3) **[Optional]** Discuss, with the help of a schematic representation in the frequency domain, what would happen to a GNSS signal received with a significant Doppler frequency and recovered by a front-end affected by I/Q imbalance.