

Assignment 2 - Solutions

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Q.1: Fourier Transform

1)

$$\begin{aligned} S_{BPSK}(f) &= \int_0^{T_c} \exp\{-j2\pi ft\} dt = \frac{1}{j2\pi f} \exp\{-j2\pi ft\}|_0^{T_c} \\ &= \frac{1}{j2\pi f} (1 - e^{-j2\pi f T_c}) = T_c \frac{\sin(\pi f T_c)}{\pi f T_c} e^{-j\pi f T_c} \\ &= T_c \text{sinc}(\pi f T_c) e^{-j\pi f T_c}. \end{aligned} \quad (1)$$

2) The Fourier Transform of $s_{1,1}(t)$ can be computed using (1) and the properties of delay property of the Fourier Transform:

$$\begin{aligned} S_{1,1}(f) &= S_{BPSK}(f/2) - S_{BPSK}(f/2) e^{-j\pi f T_c} \\ &= \frac{T_c}{2} \text{sinc}(\pi f T_c/2) e^{-j\pi f T_c/2} (1 - e^{-j\pi f T_c}) \\ &= jT_c \text{sinc}(\pi f T_c/2) \sin(\pi f T_c/2) e^{-j\pi f T_c} \\ &= jT_c \frac{\sin^2(\pi f T_c/2)}{\pi f T_c/2} e^{-j\pi f T_c}. \end{aligned} \quad (2)$$

3) Similarly, $S_{m,n}(f)$ can be computed as

$$\begin{aligned} S_{m,n}(f) &= \sum_{i=0}^{K-1} S_{1,1}\left(\frac{T_{sub}}{T_c} f\right) e^{-j\pi i T_{sub}} \\ &= S_{1,1}\left(\frac{T_{sub}}{T_c} f\right) \sum_{i=0}^{K-1} e^{-j2\pi i f T_{sub}} = S_{1,1}\left(\frac{T_{sub}}{T_c} f\right) \frac{1 - e^{-j2\pi k f T_{sub}}}{1 - e^{-j2\pi f T_{sub}}} \\ &= S_{1,1}\left(\frac{T_{sub}}{T_c} f\right) \frac{\sin(\pi f k T_{sub})}{\sin(\pi f T_{sub})} e^{-j\pi f(k-1) T_{sub}} \\ &= jT_c \frac{\sin^2(\pi f T_{sub}/2)}{\pi f T_{sub}/2} \frac{\sin(\pi f T_c)}{\frac{m}{n} \sin(\pi f T_{sub})} e^{-j\pi f T_c} \end{aligned} \quad (3)$$